**Alternate Definition of the Derivative:**

\[
f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}
\]

**Basic Derivatives**

\[
\begin{align*}
\frac{d}{dx}(x^n) &= nx^{n-1} \\
\frac{d}{dx}(\sin x) &= \cos x \\
\frac{d}{dx}(\cos x) &= -\sin x \\
\frac{d}{dx}(\tan x) &= \sec^2 x \\
\frac{d}{dx}(\cot x) &= -\csc^2 x \\
\frac{d}{dx}(\sec x) &= \sec x \tan x \\
\frac{d}{dx}(\csc x) &= -\csc x \cot x \\
\frac{d}{dx}(\ln u) &= \frac{1}{u} \frac{du}{dx} \\
\frac{d}{dx}(e^u) &= e^u \frac{du}{dx}
\end{align*}
\]

Where \( u \) is a function of \( x \), and \( a \) is a constant.

**Differentiation Rules**

**Chain Rule:**

\[
\frac{d}{dx} [f(u)] = f'(u) \frac{du}{dx} \quad \text{OR} \quad \frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}
\]

**Product Rule:**

\[
\frac{d}{dx} (uv) = u \frac{dv}{dx} + v \frac{du}{dx} \quad \text{OR} \quad u v' + v u'
\]

**Quotient Rule:**

\[
\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \quad \text{OR} \quad \frac{v u' - u v'}{v^2}
\]

**Intermediate Value Theorem**

If the function \( f(x) \) is continuous on \([a, b]\), and \( y \) is a number between \( f(a) \) and \( f(b) \), then there exists at least one number \( x = c \) in the open interval \((a, b)\) such that \( f(c) = y \).

**Mean Value Theorem**

If the function \( f(x) \) is continuous on \([a, b]\), AND the first derivative exists on the interval \((a, b)\) then there is at least one number \( x = c \) in \((a, b)\) such that

\[
f'(c) = \frac{f(b) - f(a)}{b - a}.
\]

**Rolle’s Theorem**

If the function \( f(x) \) is continuous on \([a, b]\), AND the first derivative exists on the interval \((a, b)\) AND \( f(a) = f(b) \), then there is at least one number \( x = c \) in \((a, b)\) such that \( f'(c) = 0 \).

**Extreme Value Theorem**

If the function \( f(x) \) is continuous on \([a, b]\), then the function is guaranteed to have an absolute maximum and an absolute minimum on the interval.


**Derivative of an Inverse Function:**
If $f$ has an inverse function $g$ then:

$$g'(x) = \frac{1}{f'(g(x))}$$

Derivatives are reciprocal slopes.

**Implicit Differentiation**
Remember that in implicit differentiation you will have a $\frac{dy}{dx}$ for each $y$ in the original function or equation. Isolate the $\frac{dy}{dx}$. If you are taking the second derivative $\frac{d^2y}{dx^2}$, you will often substitute the expression you found for the first derivative somewhere in the process.

**Average Rate of Change ARoC:**

$$m_{sec} = \frac{f(b) - f(a)}{b - a}$$

**Instantaneous Rate of Change IRoC:**

$$m_{tan} = f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

**Curve Sketching And Analysis**

$y = f(x)$ must be continuous at each:

**Critical point:** $\frac{dy}{dx} = 0$ or undefined


LOOK OUT FOR ENDPOINTS

**Local minimum:**

$\frac{dy}{dx}$ goes $(-, 0, +)$ or $(-, und, +)$ OR $\frac{d^2y}{dx^2} > 0$

**Local maximum:**

$\frac{dy}{dx}$ goes $(+, 0, -)$ or $(+, und, -)$ OR $\frac{d^2y}{dx^2} < 0$

**Point of inflection:** concavity changes

$\frac{d^2y}{dx^2}$ goes from $(+, 0, -), (-, 0, +), (+, und, -)$, OR $(-, und, +)$

**First Derivative:**

$f'(x) > 0$ function is increasing.

$f'(x) < 0$ function is decreasing.

$f'(x) = 0$ or DNE: Critical Values at $x$.

**Relative Maximum:** $f'(x) = 0$ or DNE and sign of $f'(x)$ changes from $+$ to $-$.  

**Relative Minimum:** $f'(x) = 0$ or DNE and sign of $f'(x)$ changes from $-$ to $+$.

**Absolute Max or Min:**
MUST CHECK ENDPOINTS ALSO

The maximum value is a $y$-value.

**Second Derivative:**

$f''(x) > 0$ function is concave up.

$f''(x) < 0$ function is concave down.

$f'(x) = 0$ and sign of $f''(x)$ changes, then there is a point of inflection at $x$.

**Relative Maximum:** $f''(x) < 0$

**Relative Minimum:** $f''(x) > 0$

**Write the equation of a tangent line at a point:**

You need a slope (derivative) and a point.

$$y_2 - y_1 = m (x_2 - x_1)$$

**Horizontal Asymptotes:**

1. If the largest exponent in the numerator is < largest exponent in the denominator then $\lim_{x \to \pm \infty} f(x) = 0$.

2. If the largest exponent in the numerator is > the largest exponent in the denominator then $\lim_{x \to \pm \infty} f(x) = DNE$

3. If the largest exponent in the numerator is = to the largest exponent in the denominator then the quotient of the leading coefficients is the asymptote.

$$\lim_{x \to \pm \infty} f(x) = \frac{a}{b}$$
The Fundamental Theorem of Calculus

\[ \frac{df}{dx} = \frac{d}{dx} \left( \int_a^x f(t) \, dt \right) = f(x) \]

PLUS A CONSTANT

The Accumulation Function

\[ F(x) = f(a) + \int_a^x f'(t) \, dt \]

The total amount, \( F(x) \), at any time \( x \), is the initial amount, \( f(a) \), plus the amount of change between \( t = a \) and \( t = x \), given by the integral.

LOGARITHMS

Definition:

\[ \ln N = p \leftrightarrow e^p = N \]

\begin{align*}
\ln e &= 1 \\
\ln 1 &= 0 \\
\ln(MN) &= \ln M + \ln N \\
\ln \left( \frac{M}{N} \right) &= \ln M - \ln N \\
p \cdot \ln M &= \ln M^p
\end{align*}

EXPONENTIAL GROWTH and DECAY:

When you see these words use: \( y = Ce^{kt} \)

"\( y \) is a differentiable function of \( t \) such that \( y > 0 \) and \( y' = ky \)"

"the rate of change of \( y \) is proportional to \( y \"

When solving a differential equation:

1. Separate variables first
2. Integrate
3. Add +C to one side
4. Use initial conditions to find "C"
5. Write the equation if the form of \( y = f(x) \)

“PLUS A CONSTANT”

The Fundamental Theorem of Calculus

\[ \int_a^b f(x) \, dx = F(b) - F(a) \]

Where \( F'(x) = f(x) \)

Corollary to FTC

\[ \frac{d}{dx} \int_a^x f(t) \, dt = f(g(u)) \frac{du}{dx} \]

FOR MORE RESOURCES VISIT:

covenantchristian.org/bird/Calculus.htm

www.teacherspayteachers.com/Store/Jean-Adams
Mean Value Theorem for Integrals:
The Average Value

If the function \( f(x) \) is continuous on \([a, b]\) and the first derivative exists on the interval \((a, b)\), then there exists a number \( x = c \) on \((a, b)\) such that

\[
\text{avg} = \frac{1}{b - a} \int_a^b f(x) \, dx = \frac{\int_a^b f(x) \, dx}{b - a}
\]

This value \( f(c) \) is the “average value” of the function on the interval \([a, b]\).

Riemann Sums

A Riemann Sum means a rectangular approximation. Approximation means that you \textbf{DO NOT EVALUATE THE INTEGRAL}; you add up the areas of the rectangles.

Trapezoidal Rule

For uneven intervals, may need to calculate area of one trapezoid at a time and total.

\[
A_{\text{trap}} = \frac{1}{2} h [b_1 + b_2]
\]

For even intervals:

\[
\int_a^b f(x) \, dx = \frac{b - a}{2n} \left[ y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n \right]
\]

Trigonometric Identities

Pythagorean Identities:

\[
sin^2 \theta + \cos^2 \theta = 1
\]

The other two are easy to derive by dividing by \( \sin^2 \theta \) or \( \cos^2 \theta \).

\[
1 + \tan^2 \theta = \sec^2 \theta
\]

\[
cot^2 \theta + 1 = \csc^2 \theta
\]

Double Angle Formulas:

\[
sin 2\theta = 2 \sin \theta \cos \theta
\]

\[
\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 1 - 2 \sin^2 \theta
\]

Power-Reducing Formulas:

\[
\cos^2 \theta = \frac{1}{2} (1 + \cos 2\theta)
\]

\[
\sin^2 \theta = \frac{1}{2} (1 - \cos 2\theta)
\]

Quotient Identities:

\[
\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}
\]

Reciprocal Identities:

\[
csc \theta = \frac{1}{\sin \theta} \quad \text{or} \quad \sin \theta \csc \theta = 1
\]

\[
\sec \theta = \frac{1}{\cos \theta} \quad \text{or} \quad \cos \theta \sec \theta = 1
\]

Values of Trigonometric Functions for Common Angles

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( \sin \theta )</th>
<th>( \cos \theta )</th>
<th>( \tan \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{\pi}{6} )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>( \frac{\sqrt{3}}{3} )</td>
</tr>
<tr>
<td>( \frac{\pi}{4} )</td>
<td>( \frac{\sqrt{2}}{2} )</td>
<td>( \frac{\sqrt{2}}{2} )</td>
<td>1</td>
</tr>
<tr>
<td>( \frac{\pi}{3} )</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>( \sqrt{3} )</td>
</tr>
<tr>
<td>( \frac{\pi}{2} )</td>
<td>1</td>
<td>0</td>
<td>&quot;( \infty )&quot;</td>
</tr>
<tr>
<td>( \pi )</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>

Must know both inverse trig and trig values:

\( \text{EX.} \quad \tan \frac{\pi}{4} = 1 \) and \( \sin^{-1} \left( \frac{1}{2} \right) = \frac{\pi}{3} \)

ODD and EVEN:

\( \sin(-x) = -\sin x \) (odd)

\( \cos(-x) = \cos x \) (even)
Area and Solids of Revolution:

NOTE: \((a, b)\) are \(x\)-coordinates and \((c, d)\) are \(y\)-coordinates

Area Between Two Curves:

Slices \(\perp\) to \(x\)-axis: \(A = \int_a^b [f(x) - g(x)] \, dx\)

Slices \(\perp\) to \(y\)-axis: \(A = \int_c^d [f(y) - g(y)] \, dy\)

Volume By Disk Method:

About \(x\)-axis: \(V = \pi \int_a^b [R(x)]^2 \, dx\)

About \(y\)-axis: \(V = \pi \int_c^d [R(y)]^2 \, dy\)

Volume By Washer Method:

About \(x\)-axis: \(V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2) \, dx\)

About \(y\)-axis: \(V = \pi \int_c^d ([R(y)]^2 - [r(y)]^2) \, dy\)

Volume By Shell Method:

About \(x\)-axis: \(V = 2 \pi \int_c^d y [R(y)] \, dy\)

About \(y\)-axis: \(V = 2 \pi \int_a^b x [R(x)] \, dx\)

General Equations for Known Cross Section
where \(base\) is the distance between the two curves and \(a\) and \(b\) are the limits of integration.

**SQUARES:** \(V = \int_a^b (base)^2 \, dx\)

**TRIANGLES**

**EQUILATERAL:** \(V = \frac{\sqrt{3}}{4} \int_a^b (base)^2 \, dx\)

**ISOSCELES RIGHT:** \(V = \frac{1}{4} \int_a^b (base)^2 \, dx\)

**RECTANGLES:** \(V = \int_a^b (base) \cdot h \, dx\)

where \(h\) is the height of the rectangles.

**SEMI-CIRCLES:** \(V = \frac{\pi}{2} \int_a^b (radius)^2 \, dx\)

where radius is \(\frac{1}{2}\) distance between the two curves.

Basic Integrals

\[
\int du = u + C
\]

\[
\int u^n du = \frac{u^{n+1}}{n+1} + C \quad n \neq -1
\]

\[
\int \frac{du}{u} = \ln|u| + C
\]

\[
e^{u} du = e^u + C
\]

\[
\int a^u du = \frac{a^u}{\ln a} + C
\]

\[
\int \sin u \, du = -\cos u + C
\]

\[
\int \cos u \, du = \sin u + C
\]

\[
\int \tan u \, du = -\ln|\cos u + C|
\]

\[
\int \cot u \, du = \ln|\sin u| + C
\]

\[
\int \sec u \, du = \ln|\sec u + \tan u| + C
\]

\[
\int \csc u \, du = -\ln|\csc u + \cot u| + C
\]

\[
\int \sec^2 u \, du = \tan u + C
\]

\[
\int \csc^2 u \, du = -\cot u + C
\]

\[
\int \sec u \tan u \, du = \sec u + C
\]

\[
\int \csc u \cot u \, du = -\csc u + C
\]

Area Between Two Curves:

Slices \(\perp\) to \(x\)-axis: \(A = \int_a^b [f(x) - g(x)] \, dx\)

Slices \(\perp\) to \(y\)-axis: \(A = \int_c^d [f(y) - g(y)] \, dy\)

Volume By Disk Method:

About \(x\)-axis: \(V = \pi \int_a^b [R(x)]^2 \, dx\)

About \(y\)-axis: \(V = \pi \int_c^d [R(y)]^2 \, dy\)

Volume By Washer Method:

About \(x\)-axis: \(V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2) \, dx\)

About \(y\)-axis: \(V = \pi \int_c^d ([R(y)]^2 - [r(y)]^2) \, dy\)

Volume By Shell Method:

About \(x\)-axis: \(V = 2 \pi \int_c^d y [R(y)] \, dy\)

About \(y\)-axis: \(V = 2 \pi \int_a^b x [R(x)] \, dx\)
**MORE DERIVATIVES:**

\[
\frac{d}{dx} \left[ \sin^{-1} \frac{u}{a} \right] = \frac{1}{\sqrt{a^2 - u^2}} \frac{du}{dx}
\]

\[
\frac{d}{dx} \left[ \tan^{-1} \frac{u}{a} \right] = \frac{a}{a^2 + u^2} \frac{du}{dx}
\]

\[
\frac{d}{dx} \left[ \sec^{-1} \frac{u}{a} \right] = \frac{a}{|u|\sqrt{u^2 - a^2}} \frac{du}{dx}
\]

\[
\frac{d}{dx} (a^u) = a^u \ln a \frac{du}{dx}
\]

\[
\frac{d}{dx} \left[ \cos^{-1} x \right] = \frac{-1}{\sqrt{1 - x^2}}
\]

\[
\frac{d}{dx} \left[ \cot^{-1} x \right] = \frac{-1}{1 + x^2}
\]

\[
\frac{d}{dx} \left[ \csc^{-1} x \right] = \frac{-1}{|x|\sqrt{x^2 - 1}}
\]

\[
\frac{d}{dx} [\log_a x] = \frac{1}{x \ln a}
\]

**MORE INTEGRALS:**

\[
\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C
\]

\[
\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C
\]

\[
\int \frac{du}{u \sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{|u|}{a} + C
\]

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<table>
<thead>
<tr>
<th>Graph</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Graph1" /></td>
<td>( y = x )</td>
</tr>
<tr>
<td><img src="image2.png" alt="Graph2" /></td>
<td>( y = x^2 )</td>
</tr>
<tr>
<td><img src="image3.png" alt="Graph3" /></td>
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</tr>
<tr>
<td><img src="image4.png" alt="Graph4" /></td>
<td>( y =</td>
</tr>
<tr>
<td><img src="image5.png" alt="Graph5" /></td>
<td>( y = \sqrt{x} )</td>
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<tr>
<td><img src="image6.png" alt="Graph6" /></td>
<td>( y = \frac{1}{x} )</td>
</tr>
<tr>
<td><img src="image7.png" alt="Graph7" /></td>
<td>( y = \sin x )</td>
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<td><img src="image8.png" alt="Graph8" /></td>
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<td>( y = \ln x )</td>
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<td>( y = \frac{1}{x^2} )</td>
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<td><img src="image12.png" alt="Graph12" /></td>
<td>( y = \sqrt{a^2 - x^2} )</td>
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