Calculus BC Chapter 10 Test 2010 Part 1 (MC)

Calculators may be used on every question. No partial credit will be given for multiple choice questions.

MULTIPLE CHOICE. It is not necessary to show your work on Multiple Choice Questions

Eliminate the parameter from the given parametric equations to find an equation that relates x and y directly.
1) \( x = 2t - 6, \ y = 6t + 3 \)
   A) \( y = 3x + 21 \) \hspace{1cm} B) \( y = \frac{1}{3}x + 6 \) \hspace{1cm} C) \( y = \frac{1}{3}x + 21 \) \hspace{1cm} D) \( y = 3x - 15 \)

2) \( x = \cos t, \ y = 4 \sin t \)
   A) \( 16x^2 + y^2 = 16 \) \hspace{1cm} B) \( 4x^2 + y^2 = 4 \) \hspace{1cm} C) \( 16x^2 - y^2 = 16 \) \hspace{1cm} D) \( x^2 + 16y^2 = 16 \)

Find \( \frac{dy}{dx} \) in terms of \( t \).
3) \( x = \frac{1}{t^6}, \ y = -3 + \ln t \)
   A) \( -\frac{t^7}{6} \) \hspace{1cm} B) \( -\frac{t^4}{6} \) \hspace{1cm} C) \( -\frac{6}{t^6} \) \hspace{1cm} D) none of these

Solve the problem.
4) Find the points at which the tangent to the curve \( x = -5 + \sin t, \ y = 4 + \cos t \) is horizontal.
   A) \((-5, 4)\) \hspace{1cm} B) \((0, 5)\) and \((\pi, 3)\) \hspace{1cm} C) \((0, 1)\) and \((0, -1)\) \hspace{1cm} D) \((-5, 5)\) and \((-5, 3)\)

5) Find the points at which the tangent to the curve \( x = 6 - t, \ y = t^3 - 5t \) is vertical.
   A) \((0, 5)\) \hspace{1cm} B) \((6, 0)\) \hspace{1cm} C) \((0, 6)\) \hspace{1cm} D) Nowhere

Find the length of the curve.
6) \( x = \frac{(20t + 35)^{3/2}}{30}, \ y = t^2 + t, \ 0 \leq t \leq 9 \)
   A) 90 \hspace{1cm} B) 135 \hspace{1cm} C) 216 \hspace{1cm} D) 378

Find the indicated vector in component form.
7) Let \( \mathbf{u} = \langle 8, 7 \rangle, \ \mathbf{v} = \langle 9, 6 \rangle \). Find \( \mathbf{u} + \mathbf{v} \).
   A) \( \langle 17, 13 \rangle \) \hspace{1cm} B) \( \langle -1, 1 \rangle \) \hspace{1cm} C) \( \langle 14, 16 \rangle \) \hspace{1cm} D) \( \langle 15, 15 \rangle \)

8) Let \( \mathbf{u} = \langle -4, 3 \rangle, \ \mathbf{v} = \langle -8, -1 \rangle \). Find \( \mathbf{u} - 6\mathbf{v} \).
   A) \( \langle -52, -3 \rangle \) \hspace{1cm} B) \( \langle -12, -12 \rangle \) \hspace{1cm} C) \( \langle -1, 5 \rangle \) \hspace{1cm} D) \( \langle 44, 9 \rangle \)

A particle travels in the plane with position vector \( \mathbf{r}(t) \). Find the velocity vector \( \mathbf{v}(t) \).
9) \( \mathbf{r}(t) = \langle e^{-7t}, \ e^{-8t} \rangle \)
   A) \( \mathbf{v}(t) = \langle e^{-7t} + 7e^{-7t}, \ 8e^{-8t} \rangle \) \hspace{1cm} B) \( \mathbf{v}(t) = \langle e^{-7t} - 7e^{-7t}, \ -8e^{-8t} \rangle \)
   C) \( \mathbf{v}(t) = \langle -7e^{-7t}, \ -8e^{-8t} \rangle \) \hspace{1cm} D) \( \mathbf{v}(t) = \langle e^{-7t} - 7e^{-7t}, \ -8e^{-8t} \rangle \)

If \( \mathbf{r}(t) \) is the position vector of a particle in the plane at time \( t \), find the particle's speed at the given value of \( t \).
10) \( \mathbf{r}(t) = \langle 6 \cos \pi t, \ 4 \sin \pi t \rangle, \ t = 1/4 \)
   A) \( 26\pi \) \hspace{1cm} B) \( \sqrt{26} \pi \) \hspace{1cm} C) \( \frac{\sqrt{26}}{2} \pi \) \hspace{1cm} D) \( \sqrt{26} \)
Calculus BC Chapter 10 Test 2010 Part 2

Calculators may be used on every question. No partial credit will be given for multiple choice questions, but you must show work to justify your answers for open response questions.

Part 1 Test Form # ______


Short Answer: Show all work needed to support your answer.

Find the slope of the polar curve at the indicated point.

11) \( r = 3 + 6 \cos \theta \), \( \theta = \frac{\pi}{2} \)

Find \( \frac{dy}{dx} \) in terms of \( t \).

12) \( x = 4 \cos t \), \( y = 5 \cos(2t) \)

Find \( \frac{d^2y}{dx^2} \) in terms of \( t \).

13) \( x = 5 \sin t \), \( y = 4 \cos t \)

Find the magnitude of the vector and the direction angle \( \theta \) it forms with the positive \( x \)-axis (express the angle in radians or degrees).

14) \( \langle -9\sqrt{3}, -9 \rangle \)

The velocity \( v(t) \) of a particle moving in the plane is given, along with the position of the particle at time \( t = 0 \). Find the position of the particle at time \( t = t_1 \).

15) \( v(t) = \langle e^t - 3t, e^t + 5t \rangle \), initial position = \( \langle 5, 2 \rangle \), \( t_1 = 4 \)
Solve the problem.

16) The velocity \( v(t) \) of a particle moving in the plane is given by the vector \( \langle 4t^2 - 3t, 2 + \cos \pi t \rangle \). Find the distance traveled by the particle from \( t = 0 \) to \( t = 4 \).

Find the area of the specified region.

17) inside one loop of the lemniscate \( r^2 = 4 \cos 2\theta \)

18) shared by the cardioids \( r = 7(1 + \sin \theta) \) and \( r = 7(1 - \sin \theta) \)

19) inside the circle \( r = -6 \sin \theta \) and outside the circle \( r = 3 \)

Solve the problem.

20) Find an expression for the length of the path traced by \( r(t) = \left( \frac{e^{2t}}{2} - 7t, e^{5t} \right) \) from \( t = 0 \) to \( t = 2 \).
Part 1

1. A
2. A
3. D
4. D
5. D
6. B
7. A
8. D
9. B
10. B
Calculus BC Chapter 10 Test 2010 Part 2

Calculators may be used on every question. No partial credit will be given for multiple choice questions, but you must show work to justify your answers for open response questions.

Part 1 Test Form #


Short Answer: Show all work needed to support your answer.

Find the slope of the polar curve at the indicated point.

11) \( r = 3 + 6 \cos \theta, \theta = \frac{\pi}{2} \)

\[
\frac{dy}{dx} = \frac{\frac{d}{d\theta} (r \sin \theta) - \frac{d}{d\theta} (3 + 6 \cos \theta) \sin \theta}{\frac{d}{d\theta} (r \cos \theta) + \frac{d}{d\theta} (3 + 6 \cos \theta) \cos \theta} = \frac{-6 \sin \theta + 6 \cos^2 \theta \cdot \frac{\cos \theta}{-3 \sin \theta} - 6 \sin \cos \theta}{-6 \sin \cos \theta - 3 \sin \cos \theta - 6 \sin \cos \theta \cdot \frac{\cos \theta}{-3 \sin \theta}}
\]

\[
\left. \frac{dy}{dx} \right|_{\theta = \frac{\pi}{2}} = \frac{-6 \cdot 0 + 0}{0 - 3 - 0} = -1
\]

Find \( \frac{dy}{dx} \) in terms of \( t \).

12) \( x = 4 \cos t, y = 5 \cos(2t) \)

\[
\frac{dy}{dx} = \frac{-2 \cdot 5 \sin(2t)}{-4 \sin t} = \frac{5 \sin(2t)}{2 \sin t}
\]

or \( 5 \cot t \)

Find \( \frac{d^2y}{dx^2} \) in terms of \( t \).

13) \( x = 5 \sin t, y = 4 \cos t \)

\[
\frac{d^2y}{dx^2} = \frac{d}{dx} \left( \frac{dy}{dx} \right) = \frac{d}{dx} \left( \frac{-4 \sin t}{5 \cos t} \right) = \frac{-4 \cos^2 t - 4 \sin^2 t}{25 \cos^3 t} = \frac{-4}{25} \sec^2 t
\]

or \( -\frac{4}{25} \sec^2 t \)

or \( -\frac{4}{25} \cot^2 t \)

Find the magnitude of the vector and the direction angle \( \theta \) it forms with the positive x-axis (express the angle in radians or degrees).

14) \((-9\sqrt{3}, -9)\)

\[
|K| = \sqrt{(-9\sqrt{3})^2 + (-9)^2} = 18 \sqrt{3} + 9 = 18 \sqrt{3} + 9
\]

\[
\theta = \arctan \left( \frac{9}{9\sqrt{3}} \right) = \frac{\pi}{6} = \frac{\pi}{6}
\]

The velocity \( v(t) \) of a particle moving in the plane is given, along with the position of the particle at time \( t = 0 \). Find the position of the particle at time \( t = 4 \).

15) \( v(t) = (e^t - 3t, e^t + 5t) \), initial position = \( (5, 2) \), \( t_1 = 4 \)

\[
r(t) = \left( 5 + \int_0^t e^{-3t} \, dt, 2 + \int_0^t e^t + 5t \, dt \right)
\]

\[
= \left( 5 + \int_0^t e^{-3t} \, dt, 2 + \int_0^t e^t + 5t \, dt \right)
\]

\[
= \left( 5 + \left[ \frac{e^{-3t}}{-3} \right]_0^4, 2 + \left[ e^t + \frac{5}{2} t^2 \right]_0^4 \right)
\]

\[
= \left( 5 - \frac{e^{-12}}{-3}, 2 + e^4 + \frac{5}{2} (4)^2 - 1 \right)
\]

\[
= \left( 5 - \frac{e^{-12}}{-3}, 2 + e^4 + \frac{5}{2} (4)^2 - 1 \right)
\]

\[
= \left( 5 - \frac{e^{-12}}{-3}, 2 + e^4 + \frac{5}{2} (4)^2 - 1 \right)
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= \left( 5 - \frac{e^{-12}}{-3}, 2 + e^4 + \frac{5}{2} (4)^2 - 1 \right)
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= \left( 5 - \frac{e^{-12}}{-3}, 2 + e^4 + \frac{5}{2} (4)^2 - 1 \right)
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= \left( 5 - \frac{e^{-12}}{-3}, 2 + e^4 + \frac{5}{2} (4)^2 - 1 \right)
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= \left( 5 - \frac{e^{-12}}{-3}, 2 + e^4 + \frac{5}{2} (4)^2 - 1 \right)
\]

\[
= \left( 5 - \frac{e^{-12}}{-3}, 2 + e^4 + \frac{5}{2} (4)^2 - 1 \right)
\]

\[
= \left( 5 - \frac{e^{-12}}{-3}, 2 + e^4 + \frac{5}{2} (4)^2 - 1 \right)
\]
Solve the problem.

16) The velocity \( v(t) \) of a particle moving in the plane is given by the vector \( (4t^2 - 3t, 2 + \cos \pi t) \). Find the distance traveled by the particle from \( t = 0 \) to \( t = 4 \).

\[
\int_0^4 \sqrt{(4t^2 - 3t)^2 + (2 + \cos \pi t)^2} \, dt = 64.186
\]

Find the area of the specified region.

17) Inside one loop of the lemniscate \( r^2 = 4 \cos 2\theta \)

\[
\text{Area} = \int_0^{\pi} \frac{1}{2} (4 \cos 2\theta)^2 \, d\theta = 34.907 \text{ or } \frac{14\pi}{2} (2\pi - \theta)
\]

18) Shared by the cardioids \( r = 7(1 + \sin \theta) \) and \( r = 7(1 - \sin \theta) \)

\[
\text{Area} = 4 \int_0^{\pi/2} \frac{1}{2} (7(1 - \sin \theta)^2) \, d\theta = \boxed{17.219} \text{ or } \frac{3}{2} (2\pi - 3\sqrt{3})
\]

19) Inside the circle \( r = -6 \sin \theta \) and outside the circle \( r = 3 \)

Solve the problem.

20) Find an expression for the length of the path traced by \( r(t) = \left( \frac{e^{2t}}{2} - 7t, e^{5t} \right) \) from \( t = 0 \) to \( t = 2 \).

\[
L = \int_0^2 \sqrt{(e^{2t} - 7)^2 + (5e^{5t})^2} \, dt
\]

\[
L = \int_0^2 \sqrt{(e^{2t} - 7)^2 + 25e^{10t}} \, dt
\]

\[
L \approx 22026.00989
\]